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2 authors:

Prachi Agrawal
National Institute of Technology, Hamirpur

Ganesh Talari
National Institute of Technology, Hamirpur

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PRACHI AGRAWAL and TALARI GANESH

Department of Mathematics
National Institute of Technology
Hamipur-177 005, Himachal Pradesh, India
E-mail: prachiagrawal202@gmail.com
ganimsc2007@gmail.com

Abstract

In Mathematical Programming, the decision makers have freedom to choose best choices to optimize each objective. This paper aims to describe the multi-choice stochastic transportation problem (MCSTP) and its solution procedure. By considering the supply and demand as parameters and treated as independent random variables which follow Logistic distribution. The cost coefficients of the objective function are in multi-choice type. The stochastic transportation problem (STP) converted to a deterministic problem and multi-objective function have converted into a single objective using binary variables. With the help of a numerical example, explained the methodology and technique for converting the multi-objective to a single objective. The main motive is to minimize the transportation cost which satisfies the requirements according to availability of the product.

Introduction

The transportation problem (TP) is the most important application of linear programming problem (Chandra, Mehra (1), Hiller (2)). In the standard transportation problem, the goal is to produce products at supply location (origins) and transferred them to demand location (destinations) at minimum cost according to the availability and requirement of the product (Hiller (2)). There are m number of origins and n number of destinations. e_k denotes the availability of the product at $k^{th}(k = 1, 2, \ldots, m)$ origin and d_l denotes the requirement of the product at $l^{th}(l = 1, 2, \ldots, n)$ destination. The

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decision variable z_{kl} describes that how many units of the product should be shipped from origin k to destination l. In the objective function, the cost coefficients c_{kl} represents the transportation cost or delivery time etc. The parameters (demand, supply, & cost) of a product are unknown to the decision maker with certainty in real life problems. The uncertainty of the parameters in the transportation problem is known as the Stochastic Transportation problem. Consider the problem

$$\text{Min } Z = \sum_{k=1}^{m} \sum_{l=1}^{n} C_{kl} z_{kl}$$

subject to

$$\Pr\left(\sum_{l=1}^{n} z_{kl} \leq e_k\right) \geq 1 - \gamma_k \;\; k = 1, 2, \ldots, m \tag{1}$$

$$\Pr\left(\sum_{k=1}^{m} z_{kl} \geq d_l\right) \geq 1 - \delta_l \;\; l = 1, 2, \ldots, n \tag{2}$$

$$z_{kl} \geq 0 \quad \forall k, \forall l, \tag{3}$$

where Pr denotes the probability of meeting a constraint. This programming problem is studied as chance constrained programming problem.

The important application (Sinha (3)) of this technique are in engineering and finance where cost, supply, demand, current exchange rate etc. are uncertain in nature. Furthermore, if the coefficients of the decision variables are multi-choice in nature, this problem is studied as Multi-Choice Stochastic Transportation Problem (MCSTP).

The Logistic distribution (Johnson, Kotz and Balakrishnan (4)) is an important in the analysis of survival data, study of income distribution and modeling of the spread of an innovation. Assume X is random variable which follow Logistic distribution with parameters μ and s (Logistic (μ, s)) (Krishnamoorthy (5)) then the probability density function (pdf) is given by

$$f_X(x) = \frac{e^{-\left(\frac{x-\mu}{s}\right)}}{s\left[1 + e^{-\left(\frac{x-\mu}{s}\right)}\right]^2} - \infty < x < \infty \tag{4}$$

$\mu > 0$ and $s > 0$ are the location and scale parameters respectively.

Dantzig (6) has developed the simplex algorithm to solve the linear programming problem. Since the transportation problem is also a kind of LPP (Ravindran, Don and James (14)), it is used for finding the optimal solution for the TP. For solving multi-choice goal programming (MCGP), Chang (7) has developed a method in which multiplicative terms of binary variables are used. He has revised the model in which the continuous variable took the place of multiplicative terms of binary variables so that MCGP converted into linear form and it can be solved by any linear programming package (Chang (8)).

Maity and Roy (9) have proposed a technique for parameters of TP such as supply, demand and cost which are considered as multi choice parameters. They have given a transformation procedure for converting multi-choice parameters into single choice using utility function. Using this approach, Mahapatra, Roy and Biswal (10) have proposed a deterministic model for MCSTP involving extreme value distribution. In 2012, they have solved MCSTP using exponential distribution. Gani and Razak (12) presented a model for two stage transportation problem in which the parameters (supply and demand) considered as fuzzy numbers. Barik, Biswal and Chakravarty (13) presented techniques for stochastic programming problem including Pareto Distribution with known mean and variance.

In this paper, presented a MCSTP with stochastic constraints involving random variables, these are defined by availability e_k and requirement d_l which follow Logistic distribution. In the subsequent sections, the stochastic constraints are converted into the deterministic constraints and by using binary variables, multi-objective function reduced into an equivalent mathematical programming. A numerical example is given to illustrate the proposed mathematical model, at the end concluding remarks are presented.

Mathematical Model

The mathematical model is constructed to convert the given multi-choice stochastic transportation problem into deterministic problem. In the given problem the constraints are in probabilistic nature such that the parameters supply and demand follows Logistic distribution and the cost coefficients are in multi choice type. MCSTP involving Logistic distribution as follows:

Model: Multi-Choice Stochastic Transportation Problem:

$$\text{Min } Z = \sum_{k=1}^{m}\sum_{l=1}^{n}\{C_{kl}^{1}, C_{kl}^{2} \ldots, C_{kl}^{t}\}z_{kl},\ t = 1, 2, \ldots, T$$

subject to

$$\Pr\left(\sum_{l=1}^{n} z_{kl} \le e_k\right) \ge 1 - \gamma_k \quad k = 1, 2, \ldots, m \tag{5}$$

$$\Pr\left(\sum_{k=1}^{m} z_{kl} \ge d_l\right) \ge 1 - \delta_l \quad l = 1, 2, \ldots, n \tag{6}$$

$$\sum_{k=1}^{m} e_k \ge \sum_{l=1}^{n} d_l \quad \text{(Feasibility Condition)} \tag{7}$$

$$z_{kl} \ge 0 \ \forall k, \forall l \tag{8}$$

where γ_k, δ_l denote the probabilities and $0 < \gamma_k < 1, 0 < \delta_l < 1$ for every k, l.

For the existence of feasible solution and to fulfill the requirement of the product at demand locations, we assumed that the total availability of the product must be greater than the requirement of the product (Equation [7] describes the feasibility condition). Here, e_k, d_l are independent random variables, follow Logistic distribution with location parameters $\mu_k, \hat{\mu}_l$ and scale parameters $s_k, \hat{s}_l$ respectively and $\{C_{kl}^{1}, C_{kl}^{2}, \ldots, C_{kl}^{l}\}$ denote the choices for cost coefficient in the objective function.

Supply $e_k (k = 1, 2, \ldots, m)$ and Demand $d_l (l = 1, 2, \ldots, n)$ follow Logistic distribution

Supply and demand follow logistic distribution then models can be formulated as:

From Equation (5) and Equation (6), the constraints are converted into its deterministic form.

$$\min Z = \sum_{k=1}^{m} \sum_{l=1}^{n} \{C_{kl}^1, C_{kl}^2, \ldots, C_{kl}^t\}\ z_{kl},\ t = 1, 2, \ldots, T$$

subject to

$$\sum_{l=1}^{n} z_{kl} \leq \mu_k - s_k \ln\left(\frac{1-\gamma_k}{\gamma_k}\right) \quad k = 1, 2, \ldots, m$$

$$\sum_{k=1}^{m} z_{kl} \geq \hat{\mu}_l - \hat{s}_l \ln\left(\frac{\delta_l}{1-\delta_l}\right) \quad l = 1, 2, \ldots, n$$

$$\sum_{k=1}^{m}\left(\mu_k - s_k \ln\left(\frac{1-\gamma_k}{\gamma_k}\right)\right) \geq \sum_{l=1}^{n}\left(\hat{\mu}_l - \hat{s}_l \ln\left(\frac{\delta_l}{1-\delta_l}\right)\right) \text{ (Feasibility Condition)}$$

Transformation Procedure

Here, it is assumed that the cost coefficients of decision variables has maximum of eight choices. By using binary variables (Maity and Roy (9)), selection of choice for the cost coefficient has been done in such a way that the whole problem is optimized. The five cases are discussed in the following manner:

Case 1. There is only one choice for the cost coefficient $(t = 1)$:

The objective function as follows:

$$Min\ Z = \sum_{k=1}^{m} \sum_{l=1}^{n} C_{kl}^1 z_{kl}$$

$$z_{kl} \geq 0 \quad \forall k, \forall l$$

C_{kl}^1 denotes the choice for the cost coefficient.

Case 2. There are two choices for the cost coefficients $(t = 2)$:

Objective function can be written as:

$$\text{Min } Z = \sum_{k=1}^{m} \sum_{l=1}^{n} \{C_{kl}^{1}, C_{kl}^{2}\} z_{kl}$$

C_{kl}^{1}, C_{kl}^{2} represent the choices for the cost coefficients and we have to choose one out of these. So that, only one binary variable is required. Let the binary variable is y_{kl}^{1}, the objective function is formulated by:

$$Min\ Z = \sum_{k=1}^{m} \sum_{l=1}^{n} \{C_{kl}^{1} y_{kl}^{1} + C_{kl}^{2}(1 - y_{kl}^{1})\} z_{kl}$$

$$z_{kl} \geq 0 \quad \forall k, \forall l$$

$$y_{kl}^{1} = 0 \text{ or } 1.$$

Case 3. There are three choices for the cost coefficients $(t = 3)$:

Objective function can be written as:

$$\text{Min } Z = \sum_{k=1}^{m} \sum_{l=1}^{n} \{C_{kl}^{1}, C_{kl}^{2}, C_{kl}^{3}\} z_{kl}.$$

$C_{kl}^{1}, C_{kl}^{2}, C_{kl}^{3}$ represent the choices for the cost coefficients and we have to choose one out of these. With the reference of Maity and Roy (9), $2 < 3 < 2^{2}$, two binary variables are required. Let these binary variable are y_{kl}^{1}, y_{kl}^{2} and additional constraints for the model are formulated by:

Model 3(i)

$$\text{Min } Z = \sum_{k=1}^{m} \sum_{l=1}^{n} \{C_{kl}^{1}(1 - y_{kl}^{1})(1 - y_{kl}^{2}) + C_{kl}^{2} y_{kl}^{1}(1 - y_{kl}^{2}) + C_{kl}^{3}(1 - y_{kl}^{1}) y_{kl}^{2}\} z_{kl}$$

$$y_{kl}^{1} + y_{kl}^{2} \leq 1$$

$$z_{kl} \geq 0 \quad \forall k, \forall l$$

$$y_{kl}^{p} = 0 \text{ or } 1 \quad p = 1, 2.$$

Model 3(ii):

$$\text{Min } Z = \sum_{k=1}^{m}\sum_{l=1}^{n}\{C_{kl}^{1}\, y_{kl}^{1}\, y_{kl}^{2} + C_{kl}^{2} y_{kl}^{1}(1 - y_{kl}^{2}) + C_{kl}^{3}(1 - y_{kl}^{1})\, y_{kl}^{2}\} z_{kl}$$

$$y_{kl}^{1} + y_{kl}^{2} \leq 1$$

$$z_{kl} \geq 0 \quad \forall k,\ \forall l$$

$$y_{kl}^{p} = 0 \text{ or } 1 \quad p = 1, 2.$$

Case 4. There are four choices for the cost coefficients $(t = 4)$:

Objective function can be written as:

$$Min\ Z = \sum_{k=1}^{m}\sum_{l=1}^{n}\{C_{kl}^{1},\ C_{kl}^{2},\ C_{kl}^{3},\ C_{kl}^{4}) z_{kl}.$$

$C_{kl}^{1},\ C_{kl}^{2},\ C_{kl}^{3},\ C_{kl}^{4}$ represent the choices for the cost coefficients and we have to choose one out of these. So, $t = 4 = 2^{2}$, two binary variables are required. Let these binary variable are $y_{kl}^{1},\ y_{kl}^{2}$ and additional constraints for the model are formulated by:

$$\text{Min } Z = \sum_{k=1}^{m}\sum_{l=1}^{n}\{C_{kl}^{1}\, y_{kl}^{1}\, y_{kl}^{2}) + C_{kl}^{2} y_{kl}^{1}(1 - y_{kl}^{2}) + C_{kl}^{3}(1 - y_{kl}^{1})\, y_{kl}^{2}$$

$$+ C_{kl}^{4}(1 - y_{kl}^{1})(1 - y_{kl}^{2})\} z_{kl}$$

$$y_{kl}^{1} + y_{kl}^{2} \leq 1$$

$$z_{kl} \geq 0 \quad \forall k,\ \forall l$$

$$y_{kl}^{p} = 0 \text{ or } 1 \quad p = 1, 2.$$

Case 5. There are five choices for the cost coefficients $(t = 5)$:

Objective function can be written as:

$$Min\ Z = \sum_{k=1}^{m}\sum_{l=1}^{n}\{C_{kl}^{1},\ C_{kl}^{2},\ C_{kl}^{3},\ C_{kl}^{4},\ C_{kl}^{5}\} z_{kl}$$

$C_{kl}^1, C_{kl}^2, C_{kl}^3, C_{kl}^4, C_{kl}^5$ represent the choices for the cost coefficients and we have to choose one out of these. So, $2^2 < 5 < 2^3$, three binary variables are required. Let these binary variable are $y_{kl}^1, y_{kl}^2, y_{kl}^3$ and additional constraints for the models are formulated as:

Model 5(i):

$$\text{Min } Z = \sum_{k=1}^{m}\sum_{l=1}^{n}\{C_{kl}^1 y_{kl}^1(1 - y_{kl}^2)(1 - y_{kl}^3) + C_{kl}^2(1 - y_{kl}^1)y_{kl}^2(1 - y_{kl}^3)$$

$$+ C_{kl}^3(1 - y_{kl}^1)(1 - y_{kl}^2)y_{kl}^3 + C_{kl}^4 y_{kl}^1 y_{kl}^2(1 - y_{kl}^3) + C_{kl}^5(1 - y_{kl}^1)y_{kl}^2 y_{kl}^3\}z_{kl}$$

$$1 \le y_{kl}^1 + y_{kl}^2 + y_{kl}^3 \le 2$$

$$y_{kl}^1 + y_{kl}^3 \le 1$$

$$z_{kl} \ge 0 \ \forall k, \forall l$$

$$y_{kl}^p = 0 \text{ or } 1 \quad p = 1, 2, 3.$$

Model 5(ii):

$$\text{Min } Z = \sum_{k=1}^{m}\sum_{l=1}^{n}\{C_{kl}^1 y_{kl}^1(1 - y_{kl}^2)(1 - y_{kl}^3) + C_{kl}^2(1 - y_{kl}^1)y_{kl}^2(1 - y_{kl}^3)$$

$$+ C_{kl}^3(1 - y_{kl}^1)(1 - y_{kl}^2)y_{kl}^3 + C_{kl}^4 y_{kl}^1 y_{kl}^2(1 - y_{kl}^3) + C_{kl}^5 y_{kl}^1(1 - y_{kl}^2)y_{kl}^3\}z_{kl}$$

$$1 \le y_{kl}^1 + y_{kl}^2 + y_{kl}^3 \le 2$$

$$y_{kl}^2 + y_{kl}^3 \le 1$$

$$z_{kl} \ge 0 \ \forall k, \forall l$$

$$y_{kl}^p = 0 \text{ or } 1 \quad p = 1, 2, 3.$$

Model 5(iii):

$$\text{Min } Z = \sum_{k=1}^{m}\sum_{l=1}^{n}\{C_{kl}^{1}\, y_{kl}^{1}(1-y_{kl}^{2})(1-y_{kl}^{3}) + C_{kl}^{2}(1-y_{kl}^{1})\, y_{kl}^{2}(1-y_{kl}^{3})$$

$$+ C_{kl}^{3}(1-y_{kl}^{1})(1-y_{kl}^{2})y_{kl}^{3} + C_{kl}^{4}(1-y_{kl}^{1})y_{kl}^{2}y_{kl}^{3} + C_{kl}^{5}(1-y_{kl}^{1})y_{kl}^{2}\, y_{kl}^{3}\}z_{kl}$$

$$1 \le y_{kl}^{1} + y_{kl}^{2} + y_{kl}^{3} \le 2$$

$$y_{kl}^{1} + y_{kl}^{2} \le 1$$

$$z_{kl} \ge 0 \ \forall k, \forall l$$

$$y_{kl}^{p} = 0 \text{ or } 1 \quad p = 1, 2, 3.$$

We have discussed up to five choices of the cost coefficients of decision variables for converting into a single choice. It can be obtained further for more choices in the same way.

An Illustration

To describe the mathematical model, considered the numerical example from the research paper (Mahapatra, Roy and Biswal (10)). The main aim is to minimize the transportation cost for transporting sea fishes from East Midnapore, West Bengal, India to the different location of India by mini-truck or etc. The sea fishes are transported from 3 supply location to 4 destination location in India through 12 routes. The transportation cost is of multi-choice type due to increasing the fuel price rate and road collection tax. The costs for every route are given in the Table 1.

Using transformation technique which is given in section 3, MCSTP is converted to single objective function. The Model is formulated as:

$$\text{Min } Z = \zeta_{11}z_{11} + \zeta_{12}z_{12} + \zeta_{13}z_{13} + \zeta_{14}z_{14} + \zeta_{21}z_{21} + \zeta_{22}z_{22}$$

$$+\zeta_{23}z_{23} + \zeta_{24}z_{24} + \zeta_{31}z_{31} + \zeta_{32}z_{32} + \zeta_{33}z_{33} + \zeta_{34}z_{34}$$

subject to

$$z_{11} + z_{12} + z_{13} + z_{14} \le 963.2390412.$$

Table 1. Transportation costs from supply location to destination location through different routes.

S.N.	Supply Location	Destination Location	Routes kl	Transportation Costs (1 unit=10 Kg)
		Kolkata	11	10 or 11 or 12
1	Mohana	Asansol	12	15 or 16
		Ranchi	13	21 or 22 or 23 or 24
		Patna	14	21 or 23 or 25
		Kolkata	21	15 or 17 or 19 or 21 or 23 or 25
2	Junput	Asansol	22	10 or 12 or 14 or 16 or 18 or 20
		Ranchi	23	9 or 10 or 11
		Patna	24	18 or 19
		Kolkata	31	20 or 21 or 22 or 23 or 24 or 25 or 26
3	Petuyaghat	Asansol	32	10 or 11 or 12 or 13 or 14 or 15 or 16 or 17
		Ranchi	33	20 or 22 or 25
		Patna	34	15 or 20

Table 2. For supplies e_k : Values of location, scale parameters with the aspiration level.

Random Variable (e_k)	Aspiration Level	Location Parameter	Scale Parameter
e_1	$\gamma_1 = 0.01$	$\mu_1 = 1000$	$s_1 = 8.0$
e_2	$\gamma_2 = 0.02$	$\mu_2 = 800$	$s_2 = 7.0$
e_3	$\gamma_3 = 0.03$	$\mu_3 = 700$	$s_3 = 6.0$

Table 3. For demands d_l : Values of location, scale parameters with the aspiration level.

Random Variable (d_l)	Aspiration Level	Location Parameter	Scale Parameter
d_1	$\delta_1 = 0.04$	$\hat{\mu}_1 = 600$	$\hat{s}_1 = 5.0$
d_2	$\delta_2 = 0.05$	$\hat{\mu}_2 = 500$	$\hat{s}_2 = 4.0$
d_3	$\delta_3 = 0.06$	$\hat{\mu}_3 = 400$	$\hat{s}_3 = 3.0$
d_4	$\delta_4 = 0.07$	$\hat{\mu}_4 = 300$	$\hat{s}_4 = 2.0$

$$z_{21}+z_{22}+z_{23}+z_{24}\le 772.7572579$$

$$z_{31}+z_{32}+z_{33}+z_{34}\le 679.1434079$$

$$z_{11}+z_{21}+z_{31}\ge 615.8902692$$

$$z_{12}+z_{22}+z_{32}\ge 511.7777559$$

$$z_{13}+z_{23}+z_{33}\ge 408.2546059$$

$$z_{14}+z_{24}+z_{34}\ge 305.1733787$$

$$\zeta_{11}=10y_{11}^{1}y_{11}^{2}+11y_{11}^{1}(1-y_{11}^{2})+12(1-y_{11}^{1})y_{11}^{2}$$

$$\zeta_{12}=15y_{12}^{1}+16(1-y_{12}^{1})$$

$$\zeta_{13}=21y_{13}^{1}y_{13}^{2}+22y_{13}^{1}(1-y_{13}^{2})+23(1-y_{13}^{1})y_{13}^{2}+24(1-y_{13}^{1})(1-y_{13}^{2})$$

$$\zeta_{14}=21y_{14}^{1}y_{14}^{2}+23y_{14}^{1}(1-y_{14}^{2})+25(1-y_{14}^{1})y_{14}^{2}$$

$$\zeta_{21}=15y_{21}^{1}(1-y_{21}^{2})(1-y_{21}^{3})+17(1-y_{21}^{1})y_{21}^{2}(1-y_{21}^{3})+19(1-y_{21}^{1})(1-y_{21}^{2})y_{21}^{3}$$

$$+21y_{21}^{1}y_{21}^{2}(1-y_{21}^{3})+23(1-y_{21}^{1})y_{21}^{2}y_{21}^{3}+25y_{21}^{1}(1-y_{21}^{2})y_{21}^{3}$$

$$\zeta_{22}=10y_{22}^{1}(1-y_{22}^{2})(1-y_{22}^{3})+12(1-y_{22}^{1})y_{22}^{2}(1-y_{22}^{3})+14y_{22}^{1}y_{22}^{2}(1-y_{22}^{3})$$

$$+16(1-y_{22}^{1})(1-y_{22}^{2})y_{22}^{3}+18y_{22}^{1}(1-y_{22}^{2})y_{22}^{3}+20(1-y_{22}^{1})y_{22}^{2}y_{22}^{3}$$

$$\zeta_{23}=9y_{23}^{1}y_{23}^{2}+10y_{23}^{1}(1-y_{23}^{2})+11(1-y_{21}^{1})y_{23}^{2}$$

$$\zeta_{24}=18y_{24}^{1}+19(1-y_{24}^{2})$$

$$\zeta_{31}=20(1-y_{31}^{1})(1-y_{31}^{2})(1-y_{31}^{3})+21y_{31}^{1}(1-y_{31}^{2})(1-y_{31}^{3})$$

$$+22(1-y_{31}^{1})y_{31}^{2}(1-y_{31}^{3})+23(1-y_{31}^{1})(1-y_{31}^{2})y_{31}^{3}$$

$$+24y_{31}^{1}y_{31}^{2}(1-y_{31}^{3})+25y_{31}^{1}(1-y_{31}^{2})y_{31}^{3}+26(1-y_{31}^{1})y_{31}^{2}y_{31}^{3}$$

$$\zeta_{32}=10y_{32}^{1}y_{32}^{2}y_{32}^{3}+11(1-y_{32}^{1})y_{32}^{3}+12y_{32}^{1}(1-y_{32}^{2})y_{32}^{3}+13y_{32}^{1}y_{32}^{2}(1-y_{32}^{3})$$

$$+14(1-y_{32}^{1})(1-y_{32}^{2})y_{32}^{3}+15y_{32}^{1}(1-y_{32}^{2})(1-y_{32}^{3})+16(1-y_{32}^{1})y_{32}^{2}(1-y_{32}^{3})$$

$$+17(1-y_{32}^{1})(1-y_{32}^{2})(1-y_{32}^{3})$$

$$\zeta_{33} = 20y_{33}^1 y_{33}^2 + 22y_{33}^1(1 - y_{33}^2) + 25(1 - y_{33}^1)y_{33}^2$$

$$\zeta_{34} = 15y_{34}^1 + 20(1 - y_{34}^2)$$

$$1 \le y_{11}^1 + y_{11}^2 \le 2$$

$$1 \le y_{14}^1 + y_{14}^2 \le 2$$

$$1 \le y_{21}^1 + y_{21}^2 + y_{21}^3 \le 2$$

$$1 \le y_{22}^1 + y_{22}^2 + y_{22}^3 \le 2$$

$$1 \le y_{23}^1 + y_{23}^2 \le 2$$

$$y_{31}^1 + y_{31}^2 + y_{31}^3 \le 2$$

$$1 \le y_{33}^1 + y_{33}^2 \le 2$$

$$z_{kl} \ge 0,\ y_{kl}^p = 0 \text{ or } 1 \quad p = 1, 2, 3,\ k = 1, 2, 3,\ l = 1, 2, 3, 4.$$

Results and Discussion

The above mathematical model for MCSTP is solved by using LINGO 11.0 software. The choices for the cost coefficients for the different routes are given in the Table 4.

Table 4. Cost coefficients for decision variables.

z_{kl}	z_{11}	z_{12}	z_{13}	z_{14}	z_{21}	z_{22}	z_{23}	z_{24}	z_{31}	z_{32}	z_{33}	z_{34}
Cost coefficients C_{kl}^t	10	15	21	25	17	10	9	18	24	11	20	15

The optimal solution for the above model is obtained as $z_{11} = 615.8903$, $z_{22} = 364.5027$, $z_{23} = 408.2546$, $z_{32} = 147.2751$, $z_{34} = 305.1734$ and rest of the decision variable are zero. The total transportation cost is Rs. 19,675.85. If all the decision variables are integers then the total transportation cost is Rs. 19,700 and for the integers variables the cost coefficients are in Table 5. The decision variables are $z_{11} = 616$, $z_{22} = 363$, $z_{23} = 409$, $z_{32} = 149$, $z_{34} = 306$.

Table 5. Cost coefficients for integer decision variables.

z_{kl}	z_{11}	z_{12}	z_{13}	z_{14}	z_{21}	z_{22}	z_{23}	z_{24}	z_{31}	z_{32}	z_{33}	z_{34}
Cost coefficients C_{kl}^{t}	10	15	23	23	15	10	9	18	22	11	22	15

Conclusion

The standard linear transportation problem solved by simplex method but, in this study, STP is converted to deterministic problem and by applying binary variables, the multi objective function reduced into single objective function. The relation $\frac{\ln(t)}{\ln(2)}$, where t is the number of choices for the cost coefficients in the objective function, describes the number of binary variables required. In the MCSTP, the parameters supply and demand treated as random variables which follow Logistic distribution and with the help of an illustration, described the mathematical model. In the example, the product is transported from 3 supply location to 4 destinations in which our main aim is to minimize the transportation cost. By using LINGO 11.0, obtained the values of the decision variables. As per Mahapatra, Roy and Biswal (10), the total transportation cost is Rs. 19891.44 as the parameters follow Extreme value distribution and by treating random variables as Logistic parameters, we got the total transportation cost Rs.19675.85 which is less.

In transportation business, MCSTP plays an important role where uncertainty always exist. This paper restricted to at best eight choices for the cost coefficients. It can be extended if there are more than eight alternatives for a parameter.

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